A Saturated Tree Network of Polling Stations with Flow Control

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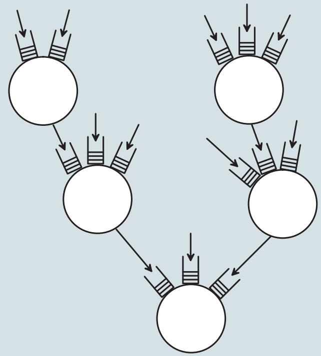
Overview

- Concentrating tree network of polling stations
- Motivation: Networks on chips
- Open network:
  - Reduction theorem
  - End-to-end delay
- Closed network (= Saturated network with flow control)
  - Division of throughput over different sources
- Summary

Concentrating tree network of polling stations



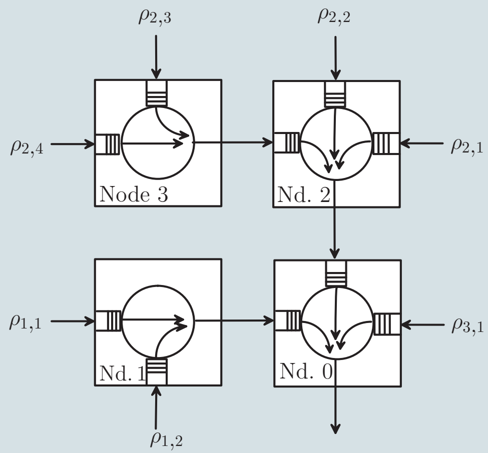
Polling station: queueing system where multiple queues are served by a single server.

Concentrating tree: Network in which all packets move towards a single node, called the sink.

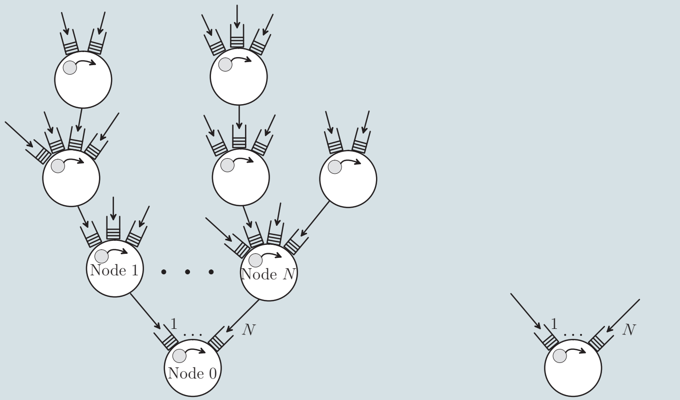
Motivation: Networks on chips

Multiple masters (e.g., processors) share a single source (e.g., memory) → all traffic has same destination.

Mesh network topology combined with XY-routing → concentrating tree network.



Open network: Reduction theorem



(a) The original system **(b)** The reduced system

Number of type-i packets (= packets passing queue i of node 0) arriving per time slot in the two models have the same distribution.

Service discipline in node 0 in both models is same HoL-based discipline.

Reduction theorem: The mean end-to-end delay (=total waiting time) of type-i packets is the same in both models.

HoL-based service disciplines

Decision which queue is served may only depend on whether or not queue are occupied and not on the number of packets in each queue.

Examples of HoL-based service disciplines:

- exhaustive service
- k-limited service
- priority

Not Hol-based service disciplines are:

- gated service
- shortest or longest queue first

Open network: End-to-end delays

The reduction theorem enables us to obtain an approximation for the mean end-to-end delay of packets per source in a tree network of polling stations in which all nodes have a service discipline that is HoL-based.

Typical assumptions:

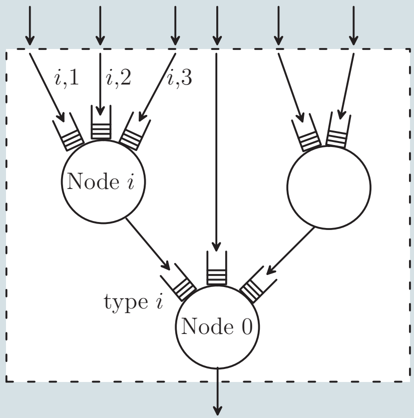
discrete-time model; deterministic service times; arrival processes are independent batch Bernoulli processes;

References

Beekhuizen, P., Denteneer, D., Resing, J. (2008).
Reduction of a polling network to a single node.
Queueing Systems **58**, 303-319.

Beekhuizen, P., Denteneer, D., Resing, J. (2008).
End-to-end delays in polling systems. *Valuetools 2008*, Athens.

Polling tree networks with flow control



Two layers of polling stations.

Question:

Division of throughput over packets from various sources, depending on flow control limits, buffer sizes, service disciplines.

Notation

Flow control limits: $L_{i,j}$

Buffer sizes at node 0: B_i
(if buffer is full blocking occurs)

Random polling service discipline at node i:

- If all queues are non-empty, queue j is served with probability $p_i(j)$.
- If some queues are empty, other queues are served with probabilities proportional to $p_i(j)$.

Throughput of type (i, j) packets: $\gamma_{i,j}$

Some trivial cases

- $B_i \geq \sum_j L_{i,j} - 1$

  In this case,

$$\gamma_{i,j} = p_0(i) \cdot \frac{L_{i,j}}{\sum_k L_{i,k}}.$$

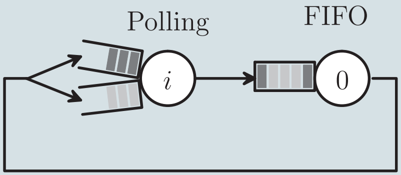
- $B_i < L_{i,j}$ for all j

  In this case,

$$\gamma_{i,j} = p_0(i) \cdot p_i(j).$$

But what if $B_i < \sum_j L_{i,j} - 1$, but not $B_i < L_{i,j}$ for all j?

If $B_i < \sum_j L_{i,j} - 1$, then $\gamma_{i,j} = p_0(i)\tilde{\gamma}_{i,j}$, where $\tilde{\gamma}_{i,j}$ is the throughput in the following closed queueing network



The state of this network can be described by $x = (x_1, x_2, \ldots, x_{B_i+1})$, where

- x_k is the type of packet in position k of the queue at node 0, for $k = 1, 2, \ldots, B_i$;
- x_{B_i+1} is the type of packet that will be served in next time slot at node i.

The equilibrium distribution of this Markov chain has the nice form

$$\pi_i(x) = C \cdot \prod_{j=1}^{N_i} \left(p_i(j)\right)^{k_j(x)}$$

where C is a normalization constant and $k_j(x)$ is the number of j's in x.

Once we have found $\pi_i(x)$, we can calculate the throughput $\tilde{\gamma}_{i,j}$ via

$$\tilde{\gamma}_{i,j} = \sum_{x:x_1=j} \pi_i(x);$$

So,

- if $B_i < L_{i,j}$ for all j, then

$$\gamma_{i,j} = p_0(i) \cdot p_i(j);$$

- if $B_i < \sum_j L_{i,j} - 1$, but not $B_i < L_{i,j}$ for all j, then

$$\gamma_{i,j} = p_0(i) \cdot \sum_{x:x_1=j} \pi_i(x);$$

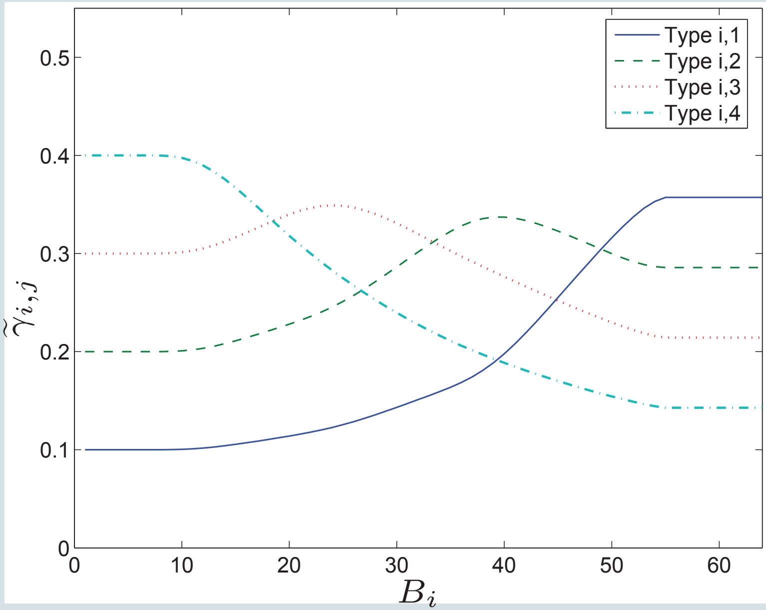
- if $B_i \geq \sum_j L_{i,j} - 1$, then

$$\gamma_{i,j} = p_0(i) \cdot \frac{L_{i,j}}{\sum_k L_{i,k}}.$$

Numerical example

Random polling service discipline at node i: $p_i = (0.1, 0.2, 0.3, 0.4)$

Flow control limits: $L_i = (20, 16, 12, 8)$



Summary

- Concentrating tree networks of polling stations
- Reduction theorem if service discipline is HoL-based
- Approximations for end-to-end delays per source if service discipline at all nodes are HoL based
- Division of throughput over different sources in model with flow control

Reference

Beekhuizen, P. (2010).
Performance Analysis of Networks on Chips.
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