Dispatching Problem with Fixed Size Jobs and Processor Sharing Discipline

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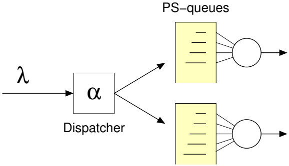
Dispatching problem to parallel queues



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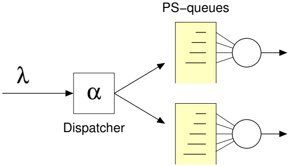

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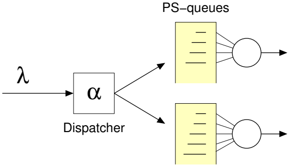

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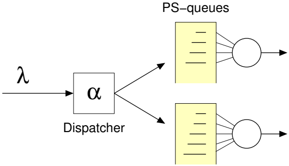

- Upon arrival a job is routed to one of the *m* servers
- Each server processes jobs according to a certain scheduling discipline (e.g., PS)
- Objective: minimize the mean delay (mean sojourn time)
- Examples:
  - job assignment in supercomputing
  - traffic routing
  - web-server farms, and
  - other distributed computing systems

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Heuristic policies

State-independent Policies:

1. **Bernoulli splitting (RND)**:
   Choose queue in random using probabilities p_i:
   i) RND-U splits the arrival stream uniformly, $p_i = 1/m$
   ii) RND-ρ balances the load, $p_i = c_i / \sum_j c_j$
   iii) RND-opt uses the p_i that minimize the mean sojourn time

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3. **Least-Work-Left (LWL)**:
   Pick the queue with the shortest backlog (Sharifnia, 1997).

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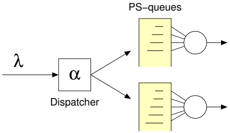
State-aware dispatching with constant job size



- Poisson arrival process, rate λ

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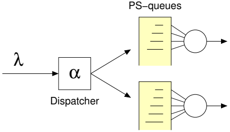

- Poisson arrival process, rate λ
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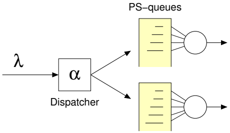

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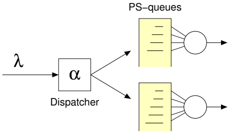

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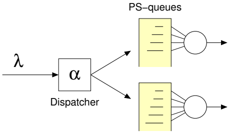

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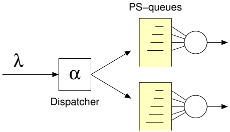

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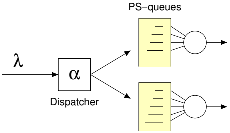

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Delay costs and relative value

Delay costs are accrued at rate

$$N_{\mathbf{z}}(t) \triangleq \text{"the number of jobs in the system"},$$

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Relative value: the expected difference in the cumulative costs between a system initially in state $\mathbf{z}$ and a system in equilibrium,

$$\begin{aligned} v_{\mathbf{z}} &\triangleq \lim_{t\to\infty} \mathrm{E}[V_{\mathbf{z}}(t) - r\, t] \\ &= \lim_{t\to\infty} \left(\mathrm{E}\Big[\int_0^t N_{\mathbf{z}}(s)\, ds\Big] - \mathrm{E}[N]\, t \right). \end{aligned}$$

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Approach: MDP and first policy iteration (FPI)

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- Requires the relative values of states $v_{\mathbf{z}}$
- However, our state-space is extremely complex (remaining service requirements at each queue)

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Decomposition to independent M/D/1-PS queues

- Deriving a relative value is generally a difficult task.

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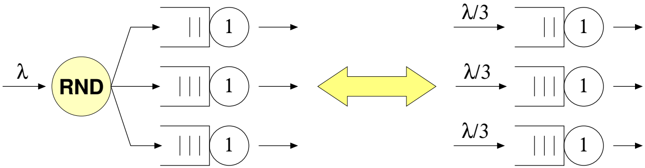
- However, any state-independent policy feeds each server jobs according to a Poisson process (cf. Bernoulli split)



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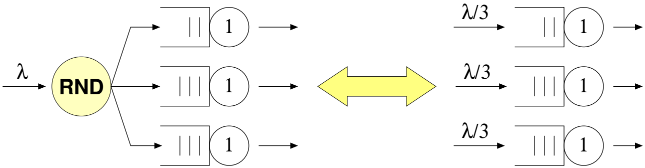

Analyze single M/D/1-PS queues instead?

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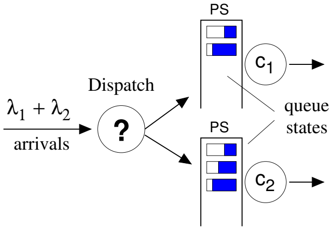
FPI of state-independent basic policy



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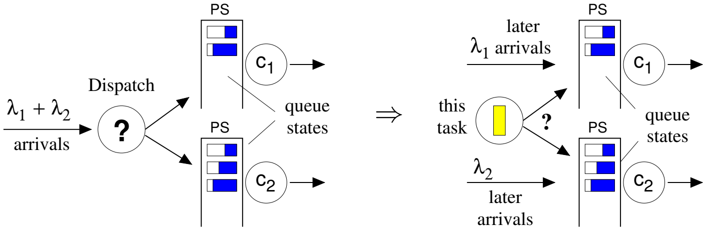

Figure: FPI considers a single decision, after which one falls back to the basic policy.

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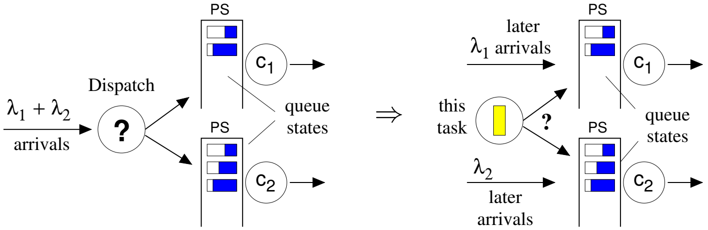

Figure: FPI considers a single decision, after which one falls back to the basic policy.

Can we solve the latter exactly?

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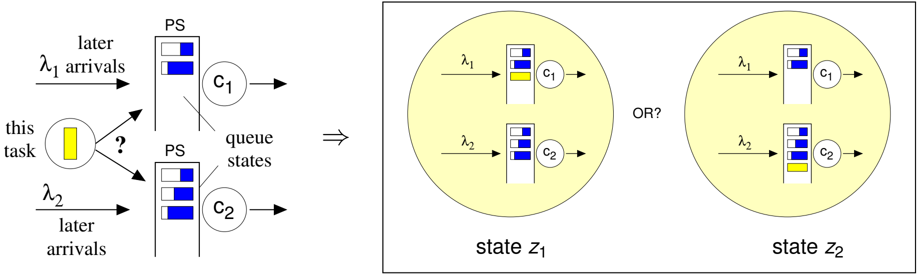

Figure: FPI of state-independent basic policy: later arrivals are dispatched according to the basic policy, isolating the queues.

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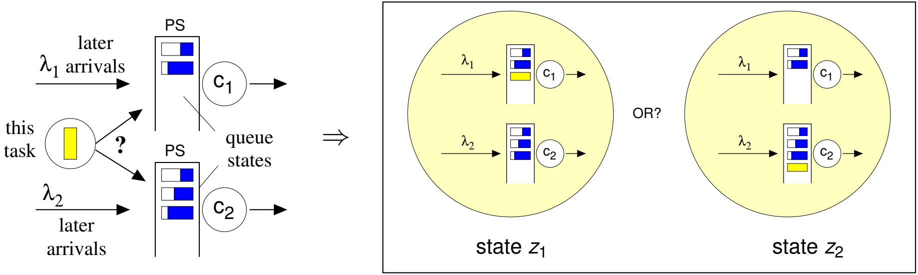

Figure: FPI of state-independent basic policy: later arrivals are dispatched according to the basic policy, isolating the queues.

The relative values v_{z_1} and v_{z_2} tell us which is the better option!

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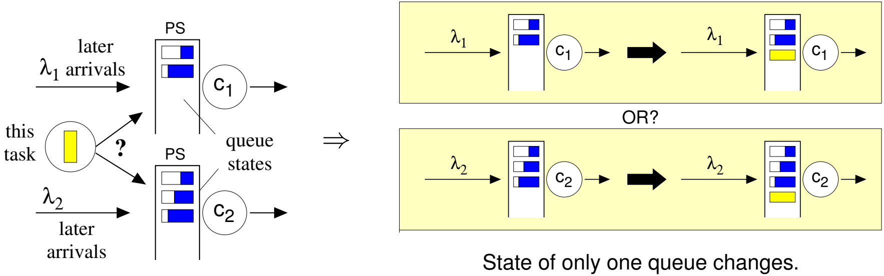

Figure: Comparison between two states in each queue.

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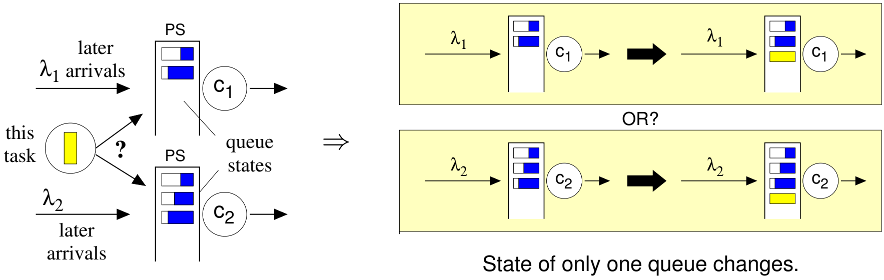

Figure: Comparison between two states in each queue.

Increments in the **queue specific relative values** $v_z^{(1)}$ and $v_z^{(2)}$ tell us which queue to choose!

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Roadmap

1. Assume a state-independent basic policy.

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4. Carry out FPI $\Rightarrow$ new efficient dispatching policy.

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In practice, it is sufficient to know, e.g., $v_{\mathbf{z}} - v_0$.

Next step: Derive $v_{\mathbf{z}} - v_0$ for an M/D/1-PS queue.

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Relative value for an M/D/1-PS queue

Notation:

- λ is the Poisson arrival rate.

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Proposition: The size-aware relative value of state **z** with respect to the delay in an M/D/1-PS queue is given by

$$v_{(\Delta_1;..;\Delta_n)} - v_0 = \boxed{\frac{\lambda}{1-\rho} u_{\mathbf{z}}^2 - u_{\mathbf{z}} + 2\sum_{i=1}^{n} i\,\Delta_i.} \qquad (1)$$

where v_0 denotes the relative value of an empty system, and $u_{\mathbf{z}} = \sum_i \Delta_i$ the backlog in the queue.

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Proof sketched

- Consider two systems under the same arrivals:
  - S1 initially in state $\mathbf{z} = (\Delta_1; ..; \Delta_n)$ with $\Delta_1 \geq \ldots \geq \Delta_n$.
  - S2 initially empty.

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- Without new arrivals, the total delay accrued in S1 is

$$\begin{aligned} \tau_{\mathbf{z}} &= \Delta_n \cdot n^2 + (\Delta_{n-1} - \Delta_n) \cdot (n-1)^2 + \ldots + (\Delta_1 - \Delta_2), \\ &= \sum_{i=1}^{n} (2i-1)\Delta_i. \end{aligned} \quad (2)$$

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- Each arrival increases the total delay (*immediate cost*)

$$s_{\mathbf{z}} = \tau_{(d;\Delta_1;..;\Delta_n)} - \tau_{(\Delta_1;..;\Delta_n)} = \boxed{2\,u_{\mathbf{z}} + d.} \quad (3)$$

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- Utilize the lack of memory of Poisson arrivals.
- Virtual busy periods similar (S1 has an offset in backlog) $\Rightarrow$ the mean contribution of a busy period.

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- Utilize the lack of memory of Poisson arrivals.
- Virtual busy periods similar (S1 has an offset in backlog)
  $\Rightarrow$ the mean contribution of a busy period.
- Details in the paper.

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Cost of a new task in M/D/1-PS

Corollary: The expected cost due to accepting a new task to an M/D/1-PS queue at state $\mathbf{z} = (\Delta_1; ..; \Delta_n)$ is given by

$$w_{\mathbf{z}} = v_{(d;\Delta_1;..;\Delta_n)} - v_{(\Delta_1;..;\Delta_n)} = \boxed{\frac{2\,u_{\mathbf{z}} + d}{1-\rho}.} \tag{4}$$

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Preemptive M/G/1-LIFO: Immediate cost in an M/G/1-LIFO is $(n + 1)x$, where x is the size of the new task.

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Preemptive M/G/1-LIFO: Immediate cost in an M/G/1-LIFO is $(n+1)x$, where x is the size of the new task. Similarly, the expected cost due to accepting a new task with size x is

$$w_{\mathbf{z}} = \boxed{\frac{(n+1)x}{1-\rho},}$$

i.e., the immediate cost divided by $1-\rho$.

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First policy iteration (FPI) with M/D/1-PS

- Assume: relative values $v_{\mathbf{z}}$ are available for basic policy

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First policy iteration (FPI) with M/D/1-PS

- Assume: relative values $v_{\mathbf{z}}$ are available for basic policy
- Improved decision according to FPI at state $\mathbf{z}$:

$$\alpha(\mathbf{z}) \triangleq \underset{i}{\text{argmin}} \left(v_{\mathbf{z}'(i)} - v_{\mathbf{z}}\right) = \underset{i}{\text{argmin}}\, w_{\mathbf{z}(i)}$$

where $\mathbf{z}'(i)$ is the new state if the job is added to queue i.

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- Assume: relative values $v_{\mathbf{z}}$ are available for basic policy
- Improved decision according to FPI at state $\mathbf{z}$:

$$\alpha(\mathbf{z}) \triangleq \underset{i}{\operatorname{argmin}} \left(v_{\mathbf{z}'(i)} - v_{\mathbf{z}} \right) = \underset{i}{\operatorname{argmin}}\, w_{\mathbf{z}(i)}$$

where $\mathbf{z}'(i)$ is the new state if the job is added to queue i.

"Choose the action with the smallest expected future cost"

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First policy iteration (FPI) with M/D/1-PS

- Assume: relative values $v_{\mathbf{z}}$ are available for basic policy
- Improved decision according to FPI at state $\mathbf{z}$:

$$\alpha(\mathbf{z}) \triangleq \underset{i}{\operatorname{argmin}} \left(v_{\mathbf{z}'(i)} - v_{\mathbf{z}}\right) = \underset{i}{\operatorname{argmin}}\, w_{\mathbf{z}(i)}$$

where $\mathbf{z}'(i)$ is the new state if the job is added to queue i.

"Choose the action with the smallest expected future cost"

- Basic policy RND-ρ balances load, $\rho_i = \rho_j$, and FPI reduces to

$$\boxed{\alpha(\mathbf{z}) = \underset{i}{\operatorname{argmin}} \left(u_i(\mathbf{z}) + 0.5\, d_i\right).}$$

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Policy family $\mathcal{P}(\beta)$

Policy family $\mathcal{P}(\beta)$ with policy parameter β is defined by

$$\operatorname*{argmin}_i \; u_i(\mathbf{z}) + \beta \cdot d_i.$$

LWL^- :	$\beta = 0$	"smallest backlog before"
LWL^+ :	$\beta = 1$	"smallest backlog afterwards"
FPI-ρ :	$\beta = 0.5$	"compromise between the above"

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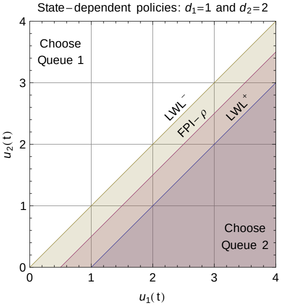
State-dependent policies in $\mathcal{P}(\beta)$ are of the switch-over type:



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Numerical examples

Performance metrics:

1. Absolute mean delay (sojourn time)
2. Relative delay when compared to FPI policy

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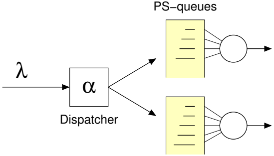
Performance metrics:

1. Absolute mean delay (sojourn time)
2. Relative delay when compared to FPI policy

Scenarios:

1. Symmetric case with two identical servers
2. Asymmetric case with two heterogeneous servers

Additionally, policy optimization within $\mathcal{P}$



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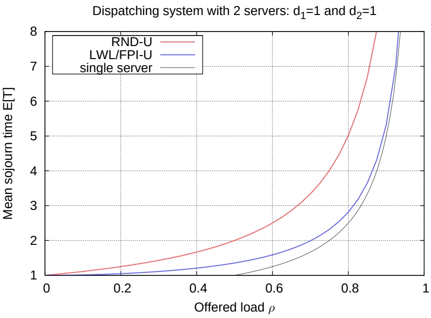
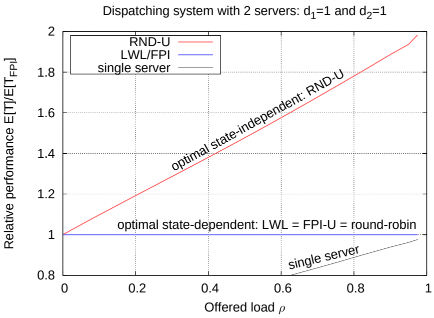
Identical servers





- Two policies, i) RND-U, ii) LWL/FPI/RR, and single server
- Left: resulting mean sojourn time
- Right: relative performance against the LWL

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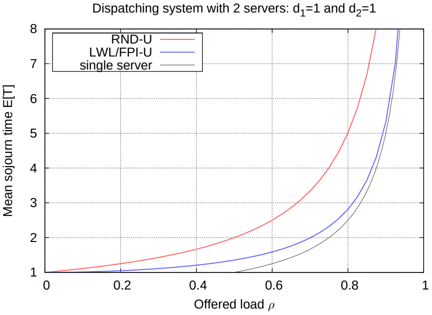
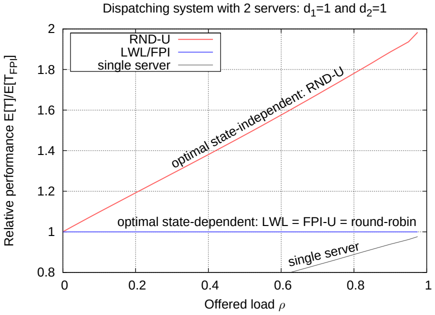
Identical servers





- Two policies, i) RND-U, ii) LWL/FPI/RR, and single server
- Left: resulting mean sojourn time
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- Optimal state-independent policy: RND-U

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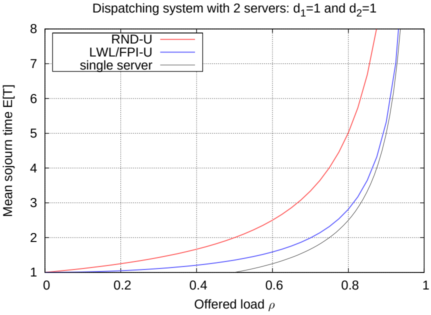
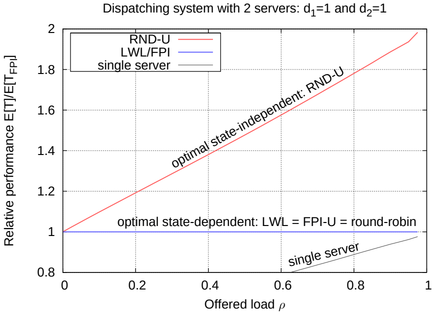
Identical servers





- Two policies, i) RND-U, ii) LWL/FPI/RR, and single server
- Left: resulting mean sojourn time
- Right: relative performance against the LWL
- Optimal state-independent policy: RND-U
- Optimal state-dependent policy: LWL/FPI-U/RR,

"Choose the queue with a smaller backlog"

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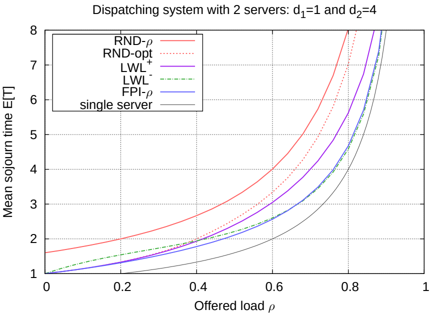
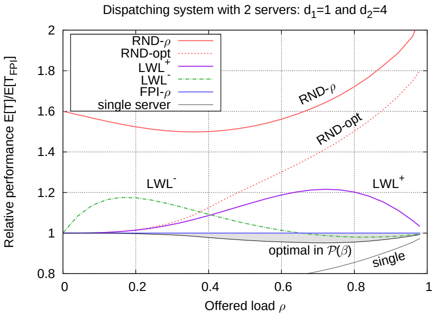
Asymmetric servers: $d_1 = 1$ and $d_2 = 4$





- Left: mean sojourn time
- Right: relative performance against the FPI-ρ policy

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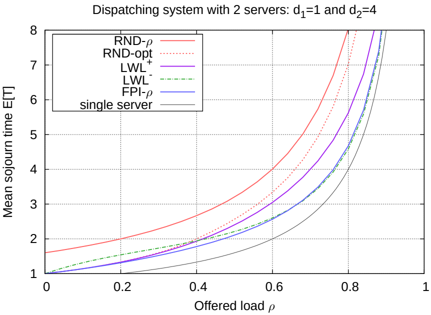
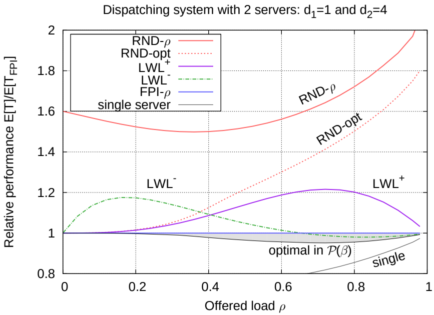
Asymmetric servers: $d_1 = 1$ and $d_2 = 4$





- Left: mean sojourn time
- Right: relative performance against the FPI-ρ policy
- Both LWL policies are clearly suboptimal

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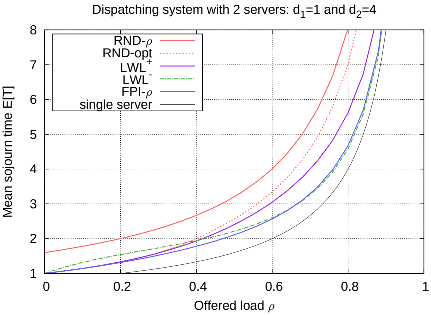

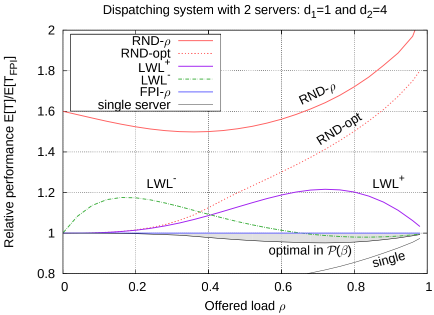

- Left: mean sojourn time
- Right: relative performance against the FPI-ρ policy
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- FPI-ρ makes very good dispatching decisions for all ρ

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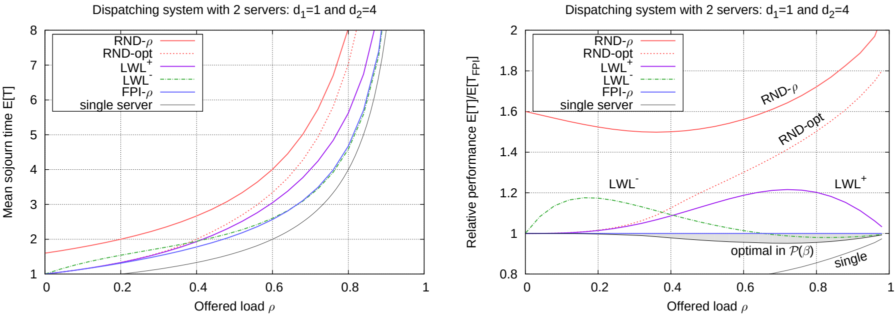
Asymmetric servers: $d_1 = 1$ and $d_2 = 4$



- Left: mean sojourn time
- Right: relative performance against the FPI-ρ policy
- Both LWL policies are clearly suboptimal
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- Gray area: optimal policy from $\mathcal{P}(\beta)$, defined by

$$u_i(\mathbf{z}) + \beta \cdot d_i.$$

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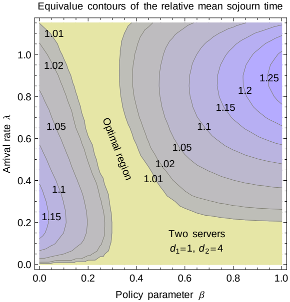
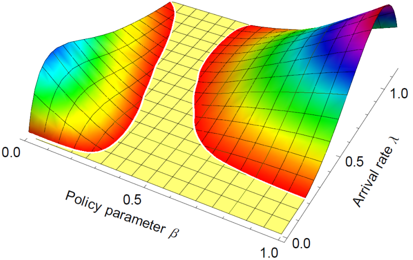
Policy optimization in $\mathcal{P}(\beta)$





- Two servers, $d_1 = 1$ and $d_2 = 4$

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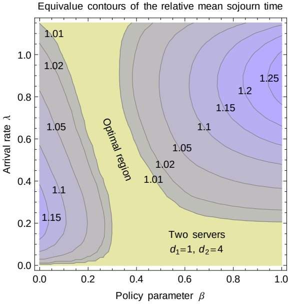
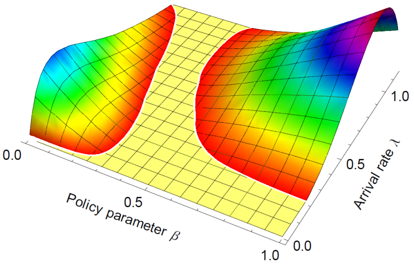
Policy optimization in $\mathcal{P}(\beta)$





- Two servers, $d_1 = 1$ and $d_2 = 4$
- x-axis: policy parameter β
  y-axis: arrival rate λ
  z-axis: mean delay relative to the optimal at given λ

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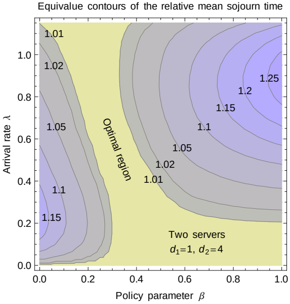

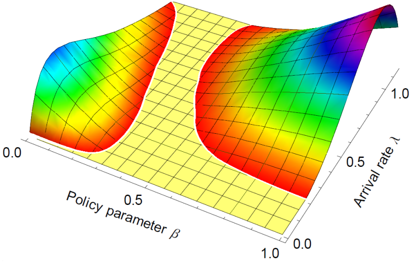

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  y-axis: arrival rate λ
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- Valley: delay is within 1% from the minimum at given λ

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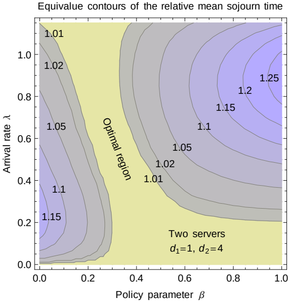

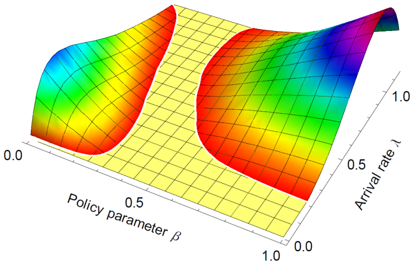

- Two servers, $d_1 = 1$ and $d_2 = 4$
- x-axis: policy parameter β
  y-axis: arrival rate λ
  z-axis: mean delay relative to the optimal at given λ
- Valley: delay is within 1% from the minimum at given λ
- FPI-ρ ($\beta = 0.5$) close to optimal optimal (within $\mathcal{P}$)

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Conclusions

- Size- and state-aware dispatching problem can be approached in MDP framework

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- General case of M/G/1-PS seems to be difficult, however, exact result for a size-aware M/M/1-PS is also available (*Hyytiä et. al, Performance 2011*)

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- For FCFS, LCFS, SPT and SRPT, the size-aware relative values are available for M/G/1 (submitted)

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Thanks!

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References:

1. E. Hyytiä, A. Penttinen and S. Aalto,
Size- and State-Aware Dispatching Problem with Queue-Specific Job Sizes,
December, 2010, (submitted).
2. E. Hyytiä, A. Penttinen, S. Aalto and J. Virtamo,
Dispatching problem with fixed size jobs and processor sharing discipline,
in 23rd International Teletraffic Congress (ITC'23), September 2011, San Fransisco, USA.
3. E. Hyytiä, J. Virtamo, S. Aalto and A. Penttinen,
M/M/1-PS Queue and Size-Aware Task Assignment,
in IFIP PERFORMANCE, October 2011, Amsterdam, Netherlands, (to appear).

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