Robustness measure for power grids with respect to cascading failures

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***Abstract*—Our daily life depends heavily on many complex infrastructures. The security of critical infrastructures has become a main issue under investigation, and therefore tools to measure the robustness of infrastructure become necessary. In this paper, we propose a new measure η to assess the robustness of the electric power grid with respect to the cascading failures. We model the power grid topology as a complex network, which consists of nodes and links representing power substations and transmission lines, respectively. The definition of the robustness measure η mainly depends on the probability of link survival as well as the depth of the cascading failures. Using the proposed metric, we compute the robustness of some realistic power grid topologies. To extend the numerical evaluation, we generate synthetic power grids with the same characteristics of the real ones to evaluate their robustness values and to compare their robustness with the real grids. We show how the depth of a cascading failure is a main factor in assessing the robustness of the power grid.**

I. INTRODUCTION

During the last decade, many electric power grids have suffered from enormous cascading failures leading to major disasters in different parts of the world. Such disasters resulted in blackouts that left millions of people without electric energy. Many types of triggers can disturb the normal functionality of the electric grid including but not limited to the voltage dips (voltage sags), brief voltage increases (swells), and transient events. In addition to the voltage faults that can harm the control devices and motor speeds, the instability of the frequency of generated voltage with large deviation may lead to asynchronization of the generators and hence, the amount of generated electric power reduces dramatically. Moreover, the weather storms and lightening may lead to shutting down some substations and damaging power transmission lines.

The main question, *how robust is the electric power grid to resist cascading failures?*, opens research areas for investigation. A cascading failure takes place when a single or multiple faults happen in the grid, and the stress on the transmission lines increases. The stress on the transmission line is the number of transmission loading relief procedures (TLR) in which the loads on the faulty lines is shifted to other lines [1].

The North American Electric Reliability Corporation (NERC) [2] introduced *Adequate Level of Reliability* (ALR) in which six characteristics of a power system are defined to guarantee a certain level of reliability under any contingency situation. NERC also introduced *Transmission System Standards: Normal and Emergency Conditions* [3] in which four different categories of events that take place in a power system are defined. The four categories are as follows:

- Category A: No Contingencies
- Category B: Event resulting in the loss of a single element.
- Category C: Event(s) resulting in the loss of two or more (multiple) elements.
- Category D: Extreme event resulting in two or more (multiple) elements removed or Cascading out of service.

These categories are used to study the relationship between the likelihood and the severity of the events as shown in [4]. Any power grid is characterized by its topology and the power flows on it. The topology represents the connectivity of substations (generation and distributions), while the power flows represent the dynamics that are controlled by the electrical characteristics of the grid, and are delivered from the generators to the distribution substations. The electrical characteristics of the grid are the capacity and the inductances of the transmission lines, the voltage values, the difference between the voltage phase angles at the terminals of each transmission line, and the loads at the distribution substations. The cascading failure takes place in the grid because some transmission lines becomes overloaded. Therefore, the robustness of the grid can not be measured only through the topology of the grid, but the power flows in the grid have to be involved as well.

In this paper, we introduce a robustness metric η of the electric grid to cascading failures. We consider the electric grid as a complex network, and the electric power represents the dynamic that flows on it. Any power grid is composed of three different functioning parts, namely, generation, transmission and distribution. The electric power flows from the generators to the distribution stations through the transmission lines. Any transmission line can experience different types of faults causing the circuit breakers at its ends to trip the line and shift the electric power flow from the faulty line to another neighbor line. If the neighbor line becomes overloaded, it

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reaches its thermal limits and becomes disconnected as well. Such a process causes a contingency situation, which may lead to cascading failure that spreads across the electric power grid. Due to the hazardous situation that an electric power grid can experience due to fault(s) in the transmission lines, we focus our studies on the transmission system network, which consists of nodes representing the power substations (buses) and links representing the transmission lines. The new metric quantifies the robustness of the electric power grid with respect to cascading failures, and it is mainly based on two essential measures, namely, the probability of link survival, and the depth of a cascading failure. The probability of link survival is the probability that a given link does not fails during a cascading failure. Therefore, it is based on the probability of link failure, which is computed through the condition that there exists a link failure in the grid and the probability that other links fail consequently, while the depth of the cascading failure represents how far the sequence of failures can reach in the grid given the start location of the failure before the grid totally collapses. Therefore, a grid that experiences a cascading failure with larger cascading depth is more robust than a grid that experience a cascading failure with shorter depth causing the power grid to collapse quickly. Also, when a cascading failure with long depth takes place in a grid, it allows the mitigation strategies to immunize the grid before a black out takes place.

Differently from the literature that do not consider the power flow, the new robustness measure relies on the power flow model to compute the flows in the transmission lines. It includes the transmission line parameters, the voltage angles and the power/load at every bus in the grid. The transmission line parameters are the inductance and the capacities. The voltage angle controls the flow direction of the electric power, and the power/ load represents the generated/consumed power at each node.

We apply the new robustness measure on three real power grid topologies given the power/load at each bus and the transmission lines parameters. To extend our numerical simulations, we generate synthetic topologies that are similar to the three real grids. The numerical results show that the power grid with large cascading depth preserves a high level of total power during a cascading failure comparing to the power grids with smaller cascading depth.

We summarize the contribution of the paper as follows:

- Proposing a new metric η to quantify the robustness of the electric power grids
- Utilizing the power flow model and the electric parameters in assessing the robustness of the grid
- Outlining the role of the link survival probability and the depth of the cascading failure

The paper is organized as follows: In Section II, we summarize the literature review. In Section III, we explain the DC power flow model in details. Next, the computational algorithm of the robustness metric is addressed in Section IV, and the power grids that are used for the numerical evaluations are presented in Section V. The numerical results are presented in Section VI, and finally the conclusions and the future work are addressed in Section VII.

II. Related work

Many works have studied the cascading failure models in the electric grid using the complex networks approach. Motter and Lai in [1] studied the cascade failures in the complex network by introducing a capacity with tolerance parameter for each node, and they assumed that every pair of nodes exchanges homogeneous flow that is routed along the shortest path connecting them. They simulated the cascading failures through random and targeted removal of nodes based on the topological characteristics and the load distribution. Their measure of robustness is the largest connected component of the network.

Lai et al. in [5] introduced an efficiency measure of the complex networks while analyzing cascading failure. The efficiency measure is inversely proportional to the shortest paths between pairs of nodes. They showed that scale-free networks are more vulnerable to attacks on short-range links than attacks on long-range links, where the range of a link connecting two nodes is the shortest path between them after removing that link. The vulnerability of the Italian GRTN power grid to the cascading failure was studied in [6] where the authors introduced an efficiency measure for each link. The link efficiency is initially homogeneous, and it is updated by the removal of a node and redistribution of the loads. The power flow between any pair of nodes follows the most efficient path separating them. They concluded that the Italian GRTN power grid is robust to many failures, while it is vulnerable to the removal of nodes with highest betweenness. Albert et al. in [7] studied the robustness of North American power grid with respect to failures leading to cascade. They presented a vulnerability measure that is proportional to the average of the normalized number of generators feeding every distribution substation. Their conclusions coincided with those in [6].

The vulnerability of the European power grid was studied in [8] and it included 24 countries. The authors found that for the removal of nodes, the reaction of power grid is similar to the reaction of scale-free networks. They concluded that the fragility of the power grid increases with the growth of the network size.

All the above methods are generic and simplified. The dynamic quantity that is carried on the complex network is routed depending on its nature and its applications. For example, the dynamic quantity in the power grid is the electric current, which flows on a link with voltage difference greater than zero. In addition, the inductances of the transmission lines govern the amount and the direction of power that flow from the power generation nodes to the distribution substations. Therefore, these methods can not reflect the robustness of the power grids since they do not consider the amount of power that flow in the grid as well as the electrical characteristics of the grid.

III. DC POWER FLOW MODEL

We consider the power grid as a complex network given that the generation, transmission substations are the nodes and the transmission lines are the links, and we denote number of nodes as N and number of links as L. The DC Power Flow model, a simplification or linearization of the full AC model, is used for the network analysis of the power grid in this work. The work [13] suggests that the term *DC Power Flow* comes from an old DC network analyzer [14], [15], in which the network branch was represented by a resistance proportional to its series reactance and each DC current was proportional to a MW flow. However, in the digital era this model became a set of simple, real (non-complex), nodal admittance matrix equations in terms of bus voltage angles and MW injections. In the AC model, the relation between real power, complex voltages and line impedance is expressed through the following equation which describes the amount of real power flowing through a transmission line [16]:

$$P_{ij} = \frac{|V_i||V_j|}{z_{ij}} sin(\delta_{ij}) \tag{1}$$

where V_i and V_j are the voltages at nodes i and j, δ_{ij} is the phase angle between these voltages and z_{ij} is the line impedance. The above equation is modified to make it suitable for the linearized analysis by making the following assumptions:

- Voltage angle differences are small, i.e. $sin(\delta_{ij}) \approx \delta_{ij}$.
- Flat Voltage profile: All voltages are considered 1 $p.u.$
- Line resistance is negligible i.e. $R << X$.

Thus, the above equation can now be written as:

$$P_{ij} = \frac{\delta_{ij}}{x_{ij}} \tag{2}$$

In terms of matrices, P is the $N \times N$ matrix of power flows between each node i and j in the network, δ is the $N \times 1$ vector of phase angles and X is the $N \times N$ weighted adjacency matrix, each element of which represents the reactance of a transmission line. It is a real number if a line is present between two nodes, and 0 otherwise.

In matrix form,

$$[P] = [b][\delta] \tag{3}$$

The matrix $[b]$ represents the imaginary part of the Y_{bus} matrix of the power grid, where $b_{ij} = -\frac{1}{x_{ij}}$ and $b_{ii} = \sum_{i \in N} -b_{ij}$ for $i \neq j$. We usually assume that there is a reference node with voltage angle equals 0.

The power handled by each node is the net sum of all the ingoing and outgoing power flows at that node as follows:

$$P_i = \sum_{j=1}^{N} P_{ij} = \sum_{j=1}^{N} (-b_{ij}\delta_{ij}) \tag{4}$$

The total load at each node is given, while the phase angles are computed using the following equation:

$$[\delta] = [b]^{-1}[P] \tag{5}$$

We assume that the power flow is below the capacity of the transmission lines [9] in normal operation. Whenever a link is removed, the power flowing through that link is redistributed to the neighboring links. These neighboring links then have to carry their own power as well as the additional power which was redistributed by the removed link. This redistribution may lead to some of the links operating beyond their capacity and failing as a result of overloading. The power from these newly failed links is again transferred to their neighbors and more failures may happen as a result. If this continues, a cascade of overloading failures may be triggered. Every time the redistribution of power takes place, all the required quantities are recomputed using the DC power flow model. This step is repeated until the cascade stops due to fragmentation of the grid causing a black out. In this paper, we assume the presence of circuit breakers to trip the overloaded transmission lines, and the absence of any mitigation strategy.

IV. COMPUTATION ALGORITHM OF THE ROBUSTNESS METRIC

We propose a new metric η to quantify the robustness of electric power grids with respect to the cascading failures. The new metric mainly depends on the probability of link survival, and on the depth of the cascading failure in the grid.

A. Probability of link survival

The probability that link l_j survives a cascade is computed as follows: We initially assume that there is a single link failure, say link l_i, and consequently, other links will fail due to the transmission loading relief procedure (TLR). Next, we identify the failed links due to the initial removal of link l_i. We repeat this process for every link l_i $i \in L$, and we list all the links that fail due to the removal of every initial link l_i. Therefore, the probability that a link l_j fails $p(l_j\ fails)$ is the total number of times that link l_j fails due to the initial removal of each link independently, divided by the overall number of failed links. Therefore, the probability of link failure reveals the most critical links in the grid. A link with the highest probability of failure is the weakest link in the grid. On the other hand, the probability of link survival $p(l_j\ survives)$ is the probability that link l_j survives a cascade, and it is equal to $1 - p(l_j\ fails)$.

B. Depth of cascading failure and the link rank

To address the depth of cascading failure, we define the link rank $r_{l_j|l_i\ removed}$ for link l_j as the cascading stage at which link l_j fails when link l_i is initially removed ($l_i\ removed$) causing the cascading failure to take place in the grid. For instance, a link, say l_j, with rank 0 ($r_{l_j|l_i\ removed}$=0) means it does not fail due to the initial removal of link l_i, while a link with rank 1 ($r_{l_j|l_i\ removed}$=1) means that the link fails in the first stage of the cascade due to the removal of link l_i and so on. The average rank of link l_j is the summation of all ranks of that link, divided by number of links L. Mathematically, the average rank of link l_j is $r_{l_j} = \frac{1}{L}\sum_{i=1}^{L} r_{l_j|l_i\ removed}$.

The robustness measure η basically depends on the probability of survival and the average link rank. Algorithm 1 introduces a computational heuristic for the robustness metric η.

Algorithm 1 Computational heuristic for the robustness measure η

```
for i = 1 to L do
  Remove link l_i
  while Cascade failure takes place do
    Recompute P, and δ
    for l_j to L do
      if P_{l_j} ≥ Capacity_{l_j} then
        Compute the rank r_{l_j|l_i removed}
        Remove link l_j
      end if
    end for
  end while
  Recreate the original power grid
end for
Compute the average rank r_{l_i}, and the survival probability
of each link p(l_i survives) ∀i ∈ L
```

Using algorithm 1, we can assess the robustness of a power grid. The product of the average rank of a link and its survival probability represents the average depth that a link can survives before the link is removed due to a cascading failure that happens in the grid. The larger the average rank, the more robust the grid is to resist the cascading failure before the power grid totally collapses causing a black out. Therefore, the robustness measure η is as follows:

$$\eta = \frac{1}{L}\sum_{j=1}^{L} p(l_j\ survives) r_{l_j} \tag{6}$$

Based on the measure in Eq. (6), we can compare the robustness of different power grids numerically. In addition, we can investigate four possible cases as follows:

1) *The probability of survival is high and the average rank is also high.*
   This is the best case. This means that a given link can survive in most of the cases when failure occurs in the grid and whenever it fails its failure happens at a much later stage of the cascade. This makes the product of the two terms higher and hence contributes to the robustness of the network.
2) *The probability of survival is high but the average rank is low.*
   In this case the link is quite robust to failures but when it fails its failure happens at an earlier stage of cascade. This causes the product of the two terms to have intermediate values. Intuitively, this case is also good for the robustness of the grid since these links can resist failure most of the times.
3) *The probability of survival is low but the average rank is high.*
   This case will again give intermediate values for the product of the two terms. However, this case is worse than the previous case because it is an indication of the presence of a weak link in the network. Although this link fails at later stages, it is almost sure that this link will fail as a result of other failures in the network.
4) *The probability of survival is low and the average rank is also low.*
   This is the worst case. This is a set of most vulnerable links in the network and these links are among the first few to fail almost always when there is a failure in the network.

V. Power grid networks

We use three different realistic power grids to evaluate the new robustness measure. For each realistic power grid, the load at each bus is given as well as the inductances of the transmission lines. The realistic grids are described as follows:

- IEEE 247 bus test system [11] with 355 links
- IEEE 118 bus test system [11] with 179 links
- WSCC 179 bus equivalent system with 222 links

Since number of available power grid topologies are very limited, we use the network generator presented in [12] to generate synthetic power grids having the same number of nodes, the same number of links, and the same maximum node degree.

VI. Numerical results

We apply the new metric η on the different power grids to measure their robustness with respect to the cascading failures. To extend the numerical simulations to larger number of grids, we generate four synthetic grids for each real power grid. We use the power flow simulator in [12], which is based on the DC power flow model, to evaluate the power flow on each link. Table I summarizes the robustness measure η of the four real grids and their synthetic ones. The table shows that the synthetic grids are generally more robust than their corresponding real grids. We also report the maximum number of cascading stages for both real and synthetic grids. We notice that the robustness measure is proportional to maximum number of cascade stages a power grid can experience. The real IEEE 247 is the most robust grid among the other real grids, while the real IEEE 118 grid is more robust than the real IEEE 179 grid.

To emphasize the role of the cascading depth in assessing the robustness of the power grids, we plot the normalized total power at each stage of the cascading failure. Figure 1 shows how the normalized total power decreases when the cascading failure takes place in the grid. We consider the case when the maximum number of cascading stages happens on each real grid. The figure shows that the IEEE 247 grid has the largest cascading depth, while it has the lowest power loss. On the opposite, the other two real grids has larger power losses and smaller cascading depth. Therefore, the IEEE 247

TABLE I
THE ROBUSTNESS MEASURE η AND THE MAXIMUM CASCADE STAGE FOR THE REAL GRIDS IEEE 247, IEEE 179 AND IEEE 118 BUS TEST SYSTEMS, AND FOUR SYNTHETIC GRIDS FOR EACH REAL GRID

Network	η	Maximum cascade stage
IEEE 247		
Real network	142.58	16
Synthetic network 1	160.03	21
Synthetic network 2	133.66	23
Synthetic network 3	138.71	26
Synthetic network 4	130.12	18
IEEE 179		
Real network	31.53	7
Synthetic network 1	114.71	15
Synthetic network 2	71.16	12
Synthetic network 3	127.31	17
Synthetic network 4	62.77	11
IEEE 118		
Real network	54.82	9
Synthetic network 1	75.42	11
Synthetic network 2	132.98	16
Synthetic network 3	138.63	17
Synthetic network 4	149.11	16

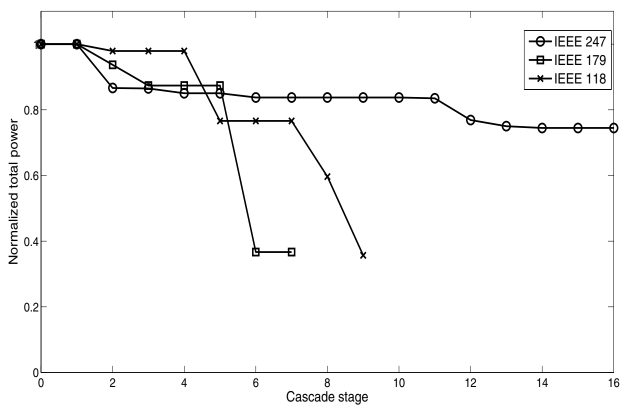

Fig. 1. Normalized total power at each cascading stage in case of the maximum cascading failures take place in the real three grids.

grid preserves a high level of total power during a cascading failure with larger depth before a total collapse takes place in the grid.

VII. CONCLUSIONS AND FUTURE WORK

We present a new measure η to assess the robustness of the power grids with respect to the cascading failures. The new measure depends on both the probability of link survival and the depth of the cascade. The probability of link survival distinguish among the fragile and strong links in the grids, and the depth of the cascade represents number of stages during which the link failures take place before the cascading failure stops due to the total collapse of the grid. Differently from the measures that are proposed in the literature, the new metric depends on evaluating the power flow in the grids given their electrical characteristics. We employ the DC power flow model to compute the power flow in the grid given the power loads and the power generation at the nodes, and the inductances of the transmission lines. The new metric is evaluated on some real and synthetic grids given their electrical characteristics. The results confirm that the robustness of the power grid is proportional to the depth of the cascading failures. Therefore, a black out event takes place in a grid with small cascading depth during the early stages of the cascading failures. In other words, a power grid with small cascading depth experiences the black out faster than a power grid with large cascading depth.

Our future work mainly focuses on extending the definition of the new measure to account for the presence of the mitigation strategies. Also, more extensive simulations will be performed on synthetic networks that are generated based on the characteristics of real power grids.

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