Planarity of Data Networks

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***Abstract*—A planar graph is one that can be drawn (say on a piece of paper) without any crossing links. In this paper we show, using a new data set, that data-network graphs show a surprisingly high likelihood of being planar. The data set — the Internet Topology Zoo — is a store of network data created from the information that network operators make public. As such, it includes meta-data that we could never have derived from automated network measurements. A surprising number of graphs in the Zoo are planar, many more than can be explained through the models for graph formation we tested. We speculate that planarity results from the requirement to build transparent networks that network operators can understand easily.**

I. Introduction

Graphs of communications networks have received a great deal of interest over the last few decades, for example [1]–[8], both through purely scientific interest and for practical reasons. Network graphs determine many of the properties of the underlying communications network such as its reliability and performance. They are therefore valuable inputs into many network algorithms, and much effort has gone into their measurement and synthesis for use in testing new algorithms.

More importantly, models of graph formation tell us something about how networks are designed. Any one engineer may be able to describe, at least loosely, their method for network design. However, the more interesting goal is to learn universal laws of network formation that may, for instance, still apply as technology evolves. Moreover, through understanding these laws we may learn how best to improve the underlying technology to fit the network design process, rather than putting the cart before the horse, as has been so often done in networking, by requiring engineers to work around technological limitations, or in providing them with features they do not need.

In this paper we note one feature – planarity – that is common in the networks we observe, but which is not explained well by the existing graph formation models. There are ongoing debates about what type of model best fits data networks: on the one hand lie the random graph models (starting with Erdos-Renyi and Gilbert [9] and going forward through Waxman [1], and more recently power-law graphs [2]–[5]). On the other hand lie "designed networks" such as the structured networks of G-ITM [6], [7] or HOT (Highly Optimized Tolerance) graphs [8]. Proponents of power-law and HOT graphs appear convincing, but both are hampered by lack of accurate data. In the few cases where a commercial network has been used the data have not been published.

There is ongoing research effort to improve the accuracy of measured networks, but we circumvent that issue entirely in this paper through a new source of network data — the Internet Topology Zoo — first described in [10]. The graphs in this dataset are derived from openly published network maps. This avoids the difficulties encountered by measurement based studies whose errors have confused the issue of topology modelling for years.

The new data is interesting, in particular, we observe a high degree of planarity in our networks. A *planar graph* can be drawn in a plane without edges crossing. Planarity has interesting consequences:

- A planar graph (without loops) is 4-colorable;
- For any planar graph we can define a dual (which is also planar), by taking one vertex in each *face* (including the outer face) and creating one edge in the dual between each face divided by an edge;
- The planar separator theorem states that every n-vertex planar graph can be partitioned into two subgraphs of size at most $2n/3$ by the removal of $O(\sqrt{n})$ vertices;

and so on. However, here we are less interested in the properties of planar graphs than we are in what this property tells us about graph formation, and how it helps us develop appropriate synthetic models for communications networks.

There are two broad approaches suggested for generating synthetic communications networks: variations on random graphs, and the structural methods most recently exemplified by optimization models [8]. The latter makes a great deal more sense when considering a computer network that will have typically been designed by a small group of engineers (rather than randomly grown). However, we use this approach to generate a large set of networks, and show that it does not generate networks with the observed degree of planarity.

This is not unexpected. The optimization approach seeks to improve some objective function usually related to the capital cost of building a network. It does not include any constraints or cost associated with link crossings. However, in real network design there is a cost for *complexity*. A more complex network is harder to manage. It is harder for a network engineer to picture. Debugging is more difficult, as there are more possible sources of errors, and the relationship between error and observations of that error may be less direct. Therefore a more complex network has a higher *operations cost*.

Operations costs associated with network designs are difficult to quantify and are, for this reason, often ignored in the

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Operations-Research literature. However, these costs are real, and are qualitatively understood by many network engineers. The result is that they often avoid purely optimized networks in favor of simple designs. We speculate that this is the cause of the high degree of planarity we observe.

Rather than discarding the existing work on network synthesis through optimization, this result implies that additional criteria should be included in such optimizations. Such criteria could crudely enforce planarity through constraints or additions to the objective function, or could be more subtle through including a "complexity" based cost, though we leave the choice of such a cost function for future research.

Finally, this paper represents a stepping stone towards a greater understanding of the explicit and implicit rules used in the construction of networks. We expect that future work in this area will use other network statistics, and other data sources to confirm and rigorously explain the phenomena observed here.

II. Data

Before we begin discussing the details of our topological data, let us first define our terminology. By topology we mean an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, which abstracts the connectivity of a data communications network. In fact, we really mean a multigraph, as multiple edges are allowed between a single pair of nodes (formally, $\mathcal{E}$ is a multiset).

Care must be taken to define the nature of the nodes and edges of the graph. Internet topologies have been given for each of the seven OSI layers: e.g., edges may refer to physical cables, virtual network layer connections, or even the HTML links between WWW pages. Other types of topology are also possible, such as those reflecting hierarchical approximations: e.g., groupings of routers into Autonomous Systems (ASs) or Points-of-Presence (PoPs). The datasets we are using contain various levels of detail, from physical fiber, through to virtual/logical connectivity between ASs. The Zoo contains various Internet communications networks, but we ensure that in each case the type of nodes and links are precisely specified.

A. The Collection Process

There are various strategies available for measuring network topology. The most direct way is to ask the network itself. IP routers are managed through *configuration files* describing the current operation of the router, and which can be used to measure a network [11]. However, these files are considered sensitive and are rarely allowed outside an organization. Thus such data may be used to construct the type of map we use here, but is otherwise rarely available to researchers.

The second class of techniques involve IP-level hacks that ideally return the path between two points. The IP header option field "record route" [13], [14] returns the route of a packet as it traverses the network, however, the more common approach is to use `traceroute` [15], [16]. Despite being commonly used, `traceroute` has many well-known deficiencies summarized in [17], [18].

There are nevertheless many studies of network topology using traceroutes (for examples see [19]–[24]), but the resulting network topologies are sometimes *very* inaccurate [17], [18], and verification against ground-truth data is difficult.

There is a third group of strategies for topology inference based on the ideas of network tomography. The statistical nature of these approaches leads to the need for verifications against difficult to obtain ground truth.

We see the Zoo data as complementary to measurement based studies. One of the potential uses of the Zoo data is to establish ground-truth data to use in testing and improving measurement-based approaches, which have, in principle, the potential to survey a much wider range of networks.

Instead of the existing automated methods we adopt here a simple, manual approach. Many companies present public material about their network, primarily for promotional purposes. They wish to sell their network.

Some care goes into such maps because they are a form of advertisement and therefore have legal requirements for accuracy; they are highly visible to potential customers; and finally, network engineers are often proud of their work, and many would very much like to display it at its best.

The most important form of published information, from our point of view, is a network map, though other supplementary data can often be very useful as well. Such maps often only show PoPs and their interconnects, but sometimes they provide much more detail. We have collected over 200 such maps and associated data, and make no claim that we have an exhaustive list. In fact it is likely that many more such maps exist, and will exist in the future. Our collection and transcription process is described in detail in [10]. In brief, we use a group of tools to aid manual transcription of the network maps. The manual process, along with the quality checks we implement, insures a highly accurate representation of the published network map is transcribed into our database. The graphs are stored in a flexible and easy to read data format — GML (the Graph Markup Language) — which allows us to include meta-data about the graph (e.g., its link capacities and node locations) and the data collection (e.g., the date of collection). GML is easily read using the Python based graph library NetworkX [25], and easily converted into other formats such as the XML derivative GraphML [26], or the *dot* format used by GraphViz [27].

The data is stored at **www.topology-zoo.org**. It is viewable through a table containing meta-data about the networks, or as a large batch file. Scripts are provided for easy access and translation of the data.

We ask that any researchers who make use of this data take care to understand the limitations of the data as documented in [10].

B. Accuracy

How accurate is the Zoo's data? The maps are created by network companies themselves, so they are based directly on ground truth. However, some network operators clearly produce these maps manually, potentially leading to inaccuracies in their depiction of their own network. There are two reasons that these errors must be much less significant than those in, for instance, traceroute studies.

- The network maps we use are all public documents, and so must satisfy standard due diligence requirements for an advertisement or official corporate publication. That is not to say that all corporations are perfect – it is easy to make mistakes in drawing the map – but a network operator is unlikely to publish a worse map than the one they use in their own network operations.
- Some network maps may idealize the network. However, we argue that in these cases, we are seeing what was in the mind of the network engineer when the network was designed. In this sense, the idealized view of the network is actually more interesting than its implementation.

It is important to spend a little time considering the second point. Many of the networks in the Zoo are not "router-level" networks. Instead, they show interconnects between Points-of-Presence (PoPs) which roughly correspond to a metropolitan area where the network operator has equipment and connectivity. It is natural that these networks are a little simpler than the router-level graphs studied in other places, but in many ways this is the part of the network design that is most interesting. This is the part that determines (to a large extent) the capital costs of links in the network (intra-PoP links are much cheaper), and the potential peering points of a network. Moreover, a PoP does not suffer the technological constraints (such as port limits) mentioned in [8] because a PoP can consist of multiple routers. So at this level, a designer has more freedom to design the desired network, rather than dealing with technological constraints that might limit the degree of a node.

A second question of accuracy is "How accurate are our transcriptions?" We have transcribed a large number of maps so it is inevitable that some errors occur. However, we have tried to minimize errors by (i) using a graphical tool so that the transcription process is closely matched to the maps, and (ii) making sure that each network is transcribed by one person, and then checked by at least one other person. The collection also has a discussion forum to allow ongoing feedback to help reduce any remaining inaccuracies. For further detailed discussion we refer the reader to [10].

C. Classification

As noted above, it is important to be precise about exactly which topology is being considered. One of the advantages of the manual transcription of the data from public information is that we can provide a number of additional classification tags for the data.

At the most basic level we classify our networks as Commercial (COM) or Research and Education Networks (REN). Our secondary type classification is related to the role the network plays: *backbone, testbed, customer, transit, access* and *internet exchange*. These are not exclusive groupings, but tags we attach to each network as appropriate. In analysing planarity we found some differences between customer and non-customer networks, and so only define that precisely here (see [10] for details of the other secondary classifications).

The *customer* tag is used when a network provided a higher level of services to its customers[1] than simple transit. We classified a network with this tag if the services provided required per-customer state: for instance, web hosting or electronic mail. With the introduction of per-customer state, the provider must have a customer service model that is not driven purely by the technical requirements of maintaining connectivity and core services (DNS, routing, etc.). This tag is applied when a provider clearly advertises a web-hosting, e-mail or co-location facility, or similar per-customer state service, for their connected organizations.

The other major aspect of network type is the layer of the network. We provide tags indicating the layer (1-3) and perhaps some more information about the type of technology being used, for instance *IP*.

We may more accurately compare networks if we focus on their area of influence [28]. The tags for this categorization are taken from the set *metro, region, country, country+, continent, continent+* and *global*.

A *metro* network is one that spans a city, or a city-sized area possibly including a small number of adjacent townships. Likewise the *country* and *continent* designations. A *region* network is approximately the size of a province, state or a small number of states, where the number of states involved is not a substantial part of the containing country. The *country+* (and *continent+*) classifications are used when the network is mostly located within one country (or continent) but has routers in another that do not correspond to a significant number of the total. The label is needed because there are many networks that are easily identified as belonging to a country (or continental) region, but for expedience have one or more routers outside the country. Where a network has significant presence in at least two continents, it is labelled a *global* network.

III. Results

A. Planarity Analysis of Zoo Maps

Datasets in the Zoo are kept in a version control system so that studies can be accurately replicated despite the ongoing nature of the data collection. The dataset used in this paper is labelled Zoo.v0.01, and contains 147 transcribed networks into the Zoo.

Some of the maps are disconnected. In these cases, we take the largest connected components. If the graph has multi-edges, we convert them (for the purpose of analyzing planarity) to single edge before performing our analysis. This step does not change the planar property of the graphs, but can change the average node degree.

Planarity was determined using the Boyer-Myrvold planarity test algorithm [29] implemented in the Matlab BGL

[1] In many cases customers might not be individual users, they may be businesses or research organizations.

(Boost Graph Library) [30]. Among the 147 networks analyzed, 21 (14%) were non-planar.

It would be natural to expect that larger, more complicated graphs are likely to be non-planar, and so we investigate the relationship between planarity, network size, and the average node degree. Figure 1 shows a scatter plot of the planar $(+)$ and non-planar (o) graphs in the Zoo. It is quite clear that the non-planar graphs tend to have higher average node degree, and that the average node degree at which the networks become commonly non-planar decreases with network size.

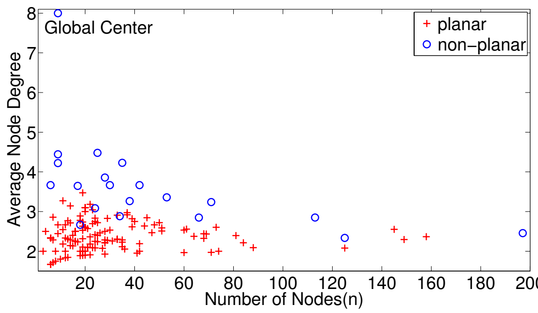


Fig. 1. Planarity of the network maps as a function of network size, and average node degree. We omit the network with 754 nodes and 895 edges whose graph is non-planar because it would not allow us to focus on the majority of smaller graphs.

Both conclusions are to be expected. For instance, there are well known upper bounds for the number of edges for a planar graph as a function of the network size. Euler's results [31] give two upper bounds: $3n-6$ for arbitrary graphs, and $2n-4$ for a graph where there are no triangles (cycles of length 3).

Network Name	Size(n)	Av. Node Degree
Airtel	9	4.2
AT&T(IP-MPLS)	25	4.48
BT(North America)	35	4.23
Chinanet	38	3.26
Cogentco	197	2.46
Dataexchange	6	3.66
Deltacom	113	2.84
Deutsche Telekom (IPTransit PoPs Only)	30	3.67
Globalcenter	9	8
Globenet	66	2.85
Goodnet	17	3.65
Gridnet	9	4.44
HurricaneElectric	24	4.6
IBM	18	2.67
IIJ	28	3.86
Ion	125	2.34
Kdl	754	2.37
Telcove	53	3.36
TW	71	3.24
UUNET	42	3.67
Xspedius	34	2.88

TABLE I
THE NON-PLANAR NETWORKS.

The 21 non-planar networks are given in Table I, where the names are attributed from the source of the data.

The breakdowns of non-planar networks according to their classifications are given in Table II. Note that some networks could not be classified. Therefore, the table elements don't necessarily sum to 147. Noteworthy points are that

- layer 1 networks are slightly more likely to be planar;
- all of the research networks are planar;
- customer networks are more likely to be non-planar; and
- larger networks (in geographic extent) are more likely to be non-planar.

However, the above results may all be conflated with network size, for instance, country-wide networks (and larger) tend to have more nodes than regional or metropolitan networks. As we expand the size of the Zoo it should be possible to do a more formal statistical analysis, controlling for the network size and degree, to understand the true factors that influence planarity.

	Classification	Planar	Non-planar
Layers	Layer1	24	3
	Layer3	90	18
Types	REN	53	0
	Commercial	73	21
Classes	Customer	37	12
	Non-Customer	36	1
Geoextent	Regional	16	1
	Country-Wide	81	13
	Country+	24	7

TABLE II
PLANARITY OF NETWORKS FOR DIFFERENT CLASSIFICATIONS.

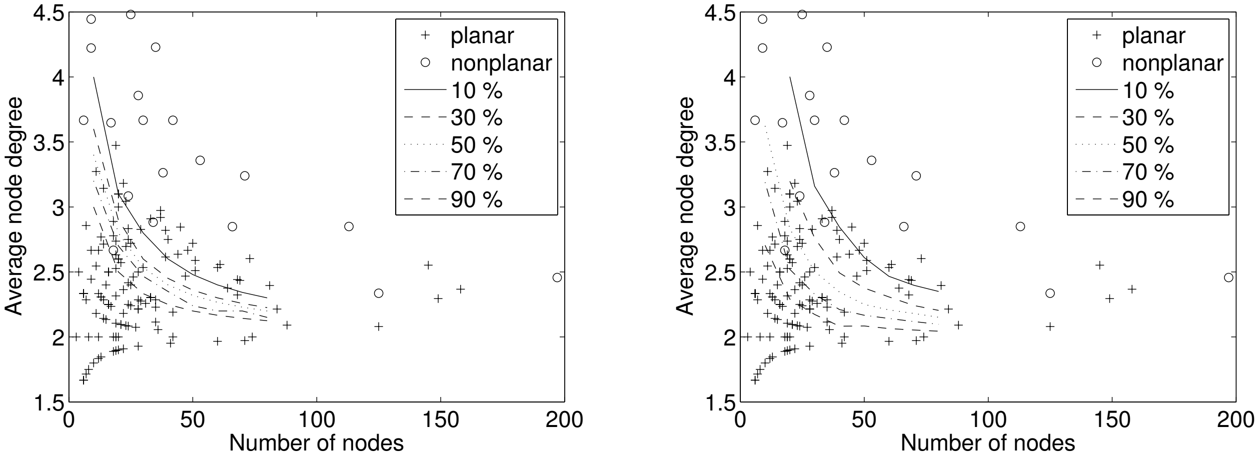
B. *Planarity of Random Graphs*

An obvious question remains. Is the degree of planarity we observe unusual, or should we expect to see these results?

There are a small set of results discussing planarity of random graphs, but these are asymptotic results for large graphs. Instead, we approach this question by generating a set of synthetic topologies of comparable size and node degree to those collected in the Zoo. From these we can gain insight into the likelihood of planarity.

Our first approach is to generate a set of Erdos-Renyi (or Gilbert) random graphs [9], and examine their planarity. An Erdos-Renyi random graph is generated by randomly choosing the node-pairs connected by a fixed number m of links (chosen to set the average node degree, which is given by $2m/n$). However, such a network is not guaranteed to be connected, so we modify it as follows. Assume that you want to generate a random connected graph with n nodes and m edges. Start by picking a node at random and calling that the *connected tree*, then pick another node at random and join it to the tree at a random position, and so on. Continue this process until all n nodes are connected. This forms the connected core of the network. After that, pick the remaining $m+1-n$ links randomly from the $(n^2-3n+2)/2$ possible node pairs that haven't been chosen yet.

For each combination of average node degree and network size we can generate a set of such networks, and measure their planarity, and from these samples derive an estimate of the



(a) Random topologies.

(b) Optimized topologies.

Fig. 2. Planarity as a function of average node degree and network size compared to contours of constant probability of planarity derived from two different network models. The percentage for each contour gives the probability of planarity along that curve. Note that we focus on the most important region of the plot which omits the KDL network with 754 nodes and 895 edges, whose graph is non-planar.

probability that a network with those parameters will be planar. However, in order to compare these results with those of the Zoo, it is more useful to have contours of constant probability for particular network size and average node degree. We construct these contours as follow: for each network size n we perform a binary search on the number of links m until we find a value that has given planarity probability p in a 95% confidence interval for 10000 trials. This procedure appears to work well within the limits of the problem (for instance, we cannot always achieve an exact match between a given probability and a particular average node degree because the number of links in a network is a discrete variable).

The contours are plotted against a scatter plot of the Zoo networks in Figure 2 (a). The contours are truncated at a maximum network size of 80 to reduce the computational cost of deriving the contours, focusing on the most interesting region.

The contours do not show a distribution function. They should be interpreted as follows. If a network lies on one of the contours, say for instance the 10% contour (near the top), then there is a one in ten probability that the network will be planar. Most networks do not sit exactly on a displayed contour but we can still estimate their probabilities from the closest contours.

The most obvious feature of Figure 2 (a) is that the number of planar graphs far exceeds what the contours predict. For example, along the 10% contour we would expect approximately 90% of the networks to be non-planar, but in fact the reverse is true – less than 10% of these graphs are non-planar. Thus, based on this simple random model, the degree of planarity observed is highly unexpected.

C. Planarity of Optimized Networks

No-one would seriously argue that Internet-like networks, such as we examine in the Zoo, are well modelled by a simple random graph of the type used above. In this section we use a recent approach for generating realistic Internet-like networks. To generate these topologies we use the idea that network topologies can be modelled as the result of an optimization problem, see for example [32]. This optimization problem is intended to model the fact that networks are designed by network engineers to fulfil a purpose, not purely as the result of a simple random process. Consequently, it is reasonable to expect that networks are optimized with respect to monetary cost and service provided, within the bounds of technological constraints. However, our approach is slightly different from [32] because we aim to be able to control the average node degree in order to construct contours.

Our approach starts with a set of node locations randomly chosen, independently and uniformly distributed over a rectangle representing the area for the network. To determine traffic demands between the nodes, each node is assigned a population independently at random from a chosen distribution (in this case exponential, although Pareto distributed populations give similar topologies). Traffic demand between a pair of nodes is then proportional to the product of the populations for those nodes, as per a standard gravity model [33].

Once the node positions and traffic have been randomly generated we derive a (near) optimal network using a genetic algorithm. The optimization has three tunable costs to allow us to obtain different types of networks:

0) A link existence cost, $C_0 = k_0$. This is a fixed cost for a link to be in the network.
1) A link length cost, $C_1 = k_1\ell$, where ℓ is the link length, determined by the positions of the two nodes at either end of the link.
2) A bandwidth-length cost, $C_2 = k_2\ell W$, where W is the link bandwidth. Required bandwidth for a link is equal to a constant multiple of the sum of the traffic demands

for the paths traversing that link (we find paths using shortest path routing based on link distances ℓ).

The total cost of a link is given by

$$C = C_0 + C_1 + C_2 = k_0 + k_1\ell + k_2\ell W.$$

We can understand how to choose the parameters k_i if we consider their impact on the final network. The network is required to be connected, so if we were to optimize only with respect to C_0 or C_1, then we would produce a minimum spanning tree. If we were to optimize only with respect to C_2, then a fully-connected network would be the optimum. We can tune between these extremes by modifying the values of the three parameters so there is a balance between the costs. In the cases presented here we do so by fixing k_0 and k_1 and changing the relative value of k_2, which allows us to control the average node degree in the resulting optimized networks. Figure 3 shows three examples of tuning the k_2 parameter for a network with 40 nodes.

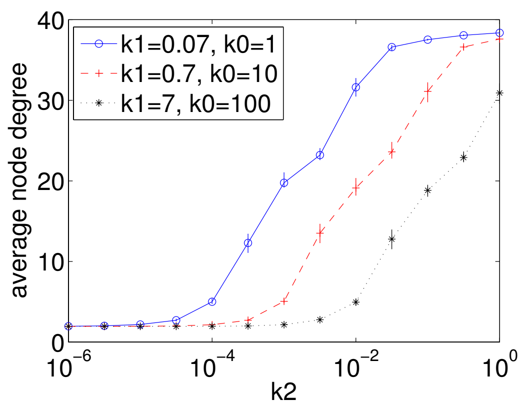


Fig. 3. Average Node Degree vs the parameter k_2. We see that varying k_2 while keeping the other parameters constant allows us fine grained control of node degree. As k_2 tends to infinity the optimal networks become fully connected, and as k_2 goes to zero, the networks tend to a tree.

For each choice of n and k_2 we generate 500 networks (from different starting node/population distributions). We measure the proportion of these that are planar as with the random graphs. We also use the bootstrap method to generate 95% confidence intervals for the estimates of the proportion that are planar. We plot the resulting estimates of the expected proportion of planar graphs in Figure 4. We perform the same analysis for networks of sizes 10, 20, 40, ..., 80 though we only plot 30 and 60 in Figure 4 for clarity.

Note that we also looked at the variation in the average node degree for the generated networks. Although the expected average node degree is set by the parameter k_2 there is some variation for particular networks as shown by the confidence interval in Figure 3. However, this variation was found to be small enough that we have not shown it in Figure 4.

The procedure for generating contours is slightly more complex here, as we cannot (within a reasonable time) generate as many samples as for the random graph. Instead, we

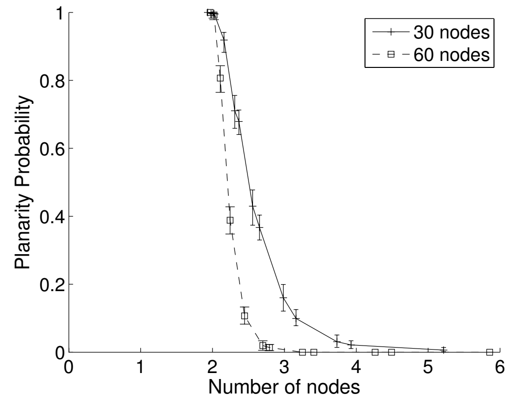


Fig. 4. Probability of planarity as a function of average node degree for optimized networks.

generate the contours by interpolating an inverted version of the curves shown in Figure 4. The resulting contours are shown in Figure 2 (b).

We can see in the figure that the contours for optimized networks are more widely separated than those for random graphs, and that, for instance, the 10% curve is higher indicating that networks can be larger and more connected (higher node degree) before non-planarity sets in. That gives a closer match to the Zoo data, but it is still very far away from explaining the high degree of planarity observed in many of the larger networks.

D. Subgraphs

We now investigate the reasons why some networks are non-planar. Kuratowski's theorem states that a graph is planar if and only if it does not contain a subgraph that is homeomorphic[2] to K_5 (the complete graph with 5 vertices) or $K_{3,3}$ (complete bipartite graph) [31]. These subgraphs are shown in Figure 5. We identify the Kuratowski subgraph, i.e., the subgraph homeomorphic K_5 or $K_{3,3}$ of our non-planar networks using the Boyer-Myrvold algorithm [29] implemented in Matlab [30]. Note that a non-planar graph can have more than one Kuratowski subgraph, but that planar graphs cannot contain any.

Table III shows the size of the Kuratowski subgraphs. In comparison to the original graph size, we see that these are 2-3 times smaller, as illustrated in Figure 6. This suggest that only a small sub-set of densely connected nodes lead to the non-planarity result.

We investigate this further by noting that tree-like structures are always planar, and that such a component cannot be the cause of non-planarity. We removed portions of the Kuratowski subgraphs that were tree-like by taking the 2-core component of the Kuratowski subgraph. We then contract the resulting graphs by removing all degree 2 nodes, effectively

[2] Two graphs are homeomorphic if there is an isomorphism from a subdivision of one to a subdivision of the other [34], where a subdivision of a graph results from inserting vertices into edges (subdivision preserves planarity).

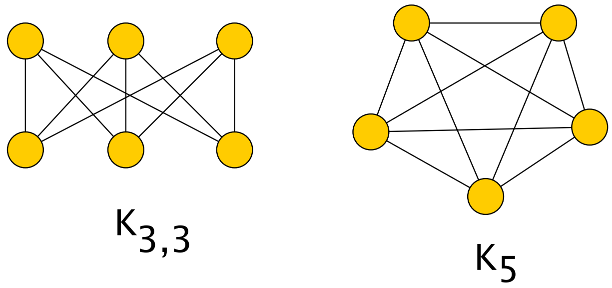


Fig. 5. The subgraphs $K_{3,3}$ and K_5 .

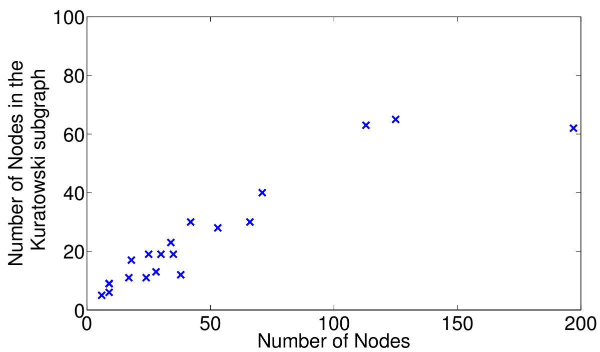


Fig. 6. Size of the Kuratowski subgraph versus the network size. We omit a network with 754 nodes and 895 edges whose graph is non-planar because it would not allow us to focus on the majority of smaller graphs. NB: there are only 19 points because 2 networks have the same values (Globalcenter and Gridnet both have 9 nodes and 9 nodes in their Kuratowski subgraphs).

reversing subdivision operation. The results are also shown in Table III. These simple operations reduced all of the networks down to less than a dozen nodes, and only a few more edges. Again, this suggests that only a very small subset of nodes are responsible for non-planarity. This may be misleading, as at present we consider one Kuratowski subgraph of each network, but it seems likely that non-planarity is unusual even in the larger, more complicated networks.

A closer look at the contraction results reveal that only 3 networks (Globalcenter, Gridnet and Dataexchange) have a Kuratowski subgraph homeomorphic to K_5 (the clique on five nodes). In the other 18 networks, their Kuratowski subgraphs contain $K_{3,3}$ subdivisions. We don't know the significance of this finding as yet, but it suggests some interesting possibilities for future research.

IV. Speculation

The results above are not definitive by any means, as we cannot rule out bias in the selection process for members of the Zoo. However the results are highly suggestive. The data at our disposal reveal an interesting, and unexpected feature of Internet networks, that is, a high likelihood of being planar. The likelihood is much higher than for the two models of network generation that we tested (it will be interesting in the future to compare other models of network generation to see if any are so likely to generate planar networks). Even in the non-planar cases, it seems that only a small subset of nodes contribute to non-planarity. We speculate that planarity does not occur by accident. It is much more likely that it is being deliberately imposed on network designs.

Why might a network engineer require planarity? Simply speaking, network engineers have to understand their network. They need to work with it to debug problems, and to manage devices. A planar graph, by virtue of being easy to draw, can be more easily understood, and therefore managed. A more complex, harder to understand network is certainly possible, but will have some cost (in terms of the engineer's time at least). These types of cost are hard to quantify and hence have often been ignored in the literature on network design and optimization. In fact, most networks are not designed using formal mathematical algorithms. They are designed "by eye". Perhaps this is in part because of the fact that formal optimization networks generate complicated, hard to understand networks.

We do not suggest that network engineers deliberately design planar networks, nor would they design for other obscure mathematical properties, but rather that planarity is one signature of the desire for simple designs.

V. Conclusion and Future Work

This paper has shown a surprising degree of planarity in observed networks. We speculated that this is caused by network designers who prefer network designs that are simple to understand. There is another potential explanation, namely that the Zoo is preferentially biased towards networks that can be drawn, because we use network maps to populate the Zoo. However, remember that non-planarity does not mean a network cannot be drawn. It only means there will be link crossings, and we observe such in many of the graphs. Even maps of planar networks often contain crossings simply because maps are usually a geographic approximation to the network, and therefore the nodes and links aren't placed arbitrarily. In any case, it is an ongoing task to better measure this property and confirm the results.

Our ideas about the cause of planarity seem intuitive, but are speculation none-the-less. We also aim to test these ideas more thoroughly through mathematical methods, e.g., by testing other types of network synthesis models to see if it arises naturally through other means, or through looking more carefully for the sources of non-planarity in graphs. We can also look for other signatures of "simplicity" in network design, to see if the underlying hypothesis, about how network engineers work, is justified.

As a network evolves, and grows, engineers may not mathematically optimize the network, but they certainly do try to improve it. So the optimization paradigm for network synthesis should not be discarded as a result of this work. We need, however, to consider the effect of more complicated optimization objectives, which penalize "complex" designs. However, it is not at all clear how to create such an objective, or optimize a network against it, and so this is yet another topic for future research.

Network Name	Kuratowski subgraph		2-core		Contracting degree 2 nodes	
	# of nodes	#of edges	# of nodes	# of edges	# of nodes	# of edges
Airtel	6	9	6	9	6	9
AT&T(IP-MPLS)	19	22	13	16	8	11
BT(North America)	19	22	16	19	7	10
Chinanet	12	15	10	13	8	11
Cogentco	62	65	25	28	8	11
Dataexchange	5	10	5	10	5	10
Deltacom	63	66	31	34	8	11
Deutsche Telekom (IPTransit PoPs Only)	19	22	14	17	9	12
Globalcenter	9	14	6	11	6	11
Globenet	30	33	21	24	8	11
Goodnet	11	14	11	14	7	10
Gridnet	9	14	9	14	5	10
HurricaneElectric	11	14	10	13	7	10
IBM	17	20	14	17	8	11
IIJ	13	16	11	14	8	11
Ion	65	68	27	30	8	11
Kdl	263	266	202	205	11	14
Telcove	28	31	27	30	8	11
TW	40	43	29	32	9	12
UUNET	30	33	22	25	7	10
Xspedius	23	26	20	23	7	10

TABLE III
KURATOWSKI SUBGRAPHS OF NON-PLANAR GRAPHS.

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